

Березин Сергей Викторович,
Электрик, «МБУ»,
РФ, Респ. Башкортостан, с. Иглино
<https://orcid.org/0000-0001-8086-8288>

Berezin Sergey Viktorovich,
Electrician, "MBU",
RF, r. Bashkortostan, s. Iglino
<https://orcid.org/0000-0001-8086-8288>

**МЕТОД MASTER-J: НАХОЖДЕНИЕ ТОЧНОГО
КОНЕЧНОГО МНОЖЕСТВА КОРНЕЙ ТРЕХЧЛЕНОВ
С КОМПЛЕКСНЫМИ СТЕПЕНЯМИ**
**THE MASTER-J METHOD: FINDING THE EXACT FINITE SET
OF ROOTS OF TRINOMIALS WITH COMPLEX EXPONENTS**

Аннотация: Мы представляем аналитический метод (метод Master-J) для нахождения корней алгебраических уравнений с комплексными степенями, который позволяет:

1. Определить, имеет ли уравнение конечное или бесконечное число корней.
2. Найти все корни, когда их число конечно.
3. Предоставить явные выражения для каждого корня.
4. Описать структуру ветвления корней в комплексной плоскости, когда число корней бесконечно.

Abstract: We present an analytic method (the Master-J method) for finding the roots of an algebraic equation with complex exponents, enabling:

1. Determination of whether the equation has a finite or infinite number of roots.
2. Finding all roots when their number is finite.
3. Providing explicit expressions for each root.
4. Providing a description of the branching structure of the roots in the complex plane when the number of roots is infinite.

Ключевые слова: Master-J, мастер-ряд, аналитические решения, степенные ряды, комплексные степени.

Key words: Master-J, master series, analytical solutions, power series, complex degrees.

1. INTRODUCTION

Until now, there has been no general analytical method for solving equations with complex exponents. This fundamental limitation equally applies to equations with irrational exponents and to algebraic equations of degree five and higher, where traditional algebraic methods fail.

This work introduces the Master-J method, which for the first time provides a unified analytical framework for solving equations with integer, irrational, and complex exponents. The method is based on the construction of "master series" - power series whose coefficients are expressed through generalized factorials called "master numbers". The key advantage of the method is its ability to systematically determine the solution set: divergence of the series for a specific branch index always indicates the absence of a root for that branch, while convergence indicates a potential root that requires verification through the principal branch of the complex logarithm.

Using the example of trinomial equations with complex exponents whose real parts are distinct and non-zero, it is shown that the Master-J method guarantees finding all roots and proves



the absence of other solutions. The method demonstrates that in many cases (particularly when the real parts of the exponents are distinct and non-zero) the number of roots is finite, and this fact can be established analytically. Thus, the method eliminates a fundamental gap associated with the lack of tools for rigorous analysis of such equations.

The work is organized as follows: Section 2 briefly describes the apparatus of master numbers and master series; Section 3 demonstrates the application of the method to specific equations with complex exponents; In Section 4 we conclude and discuss the implications and prospects of the method's application.

2. THE MASTER-J METHOD APPARATUS

We start by defining the fundamental objects of the method: "master number" and "master series". A master number is defined by the formula:

$$\text{mstr}(m; s; r; t) := \frac{m!s}{r!t} = \prod_{j=1}^{t-1} (m + st - rj) \quad (1)$$

where $\{m, s, r\}$ are complex parameters and, by definition, $\frac{m!s}{r!1} = 1$. A master series is defined as the expansion:

$$\text{msr}(m; s; r; x) := \frac{m!s}{r!x} = m + \sum_{t=1}^{\infty} \left(\frac{x^t}{t!} \cdot \frac{m!s}{r!t} \right) \quad (2)$$

The following are important identities that follow from the above definitions:

$$\frac{1!s}{r!x} = e^{\frac{0!s}{r!x}} \quad (3)$$

$$\frac{d \frac{1!s}{r!x}}{dx} = \frac{1!s}{r!x} \frac{d \frac{0!s}{r!x}}{dx} \quad (4)$$

$$\frac{1!s}{r!x} = \frac{d \frac{1!s}{r!x}}{d \ln \frac{1!s}{r!x}} = \frac{d e^{\frac{0!s}{r!x}}}{d \frac{0!s}{r!x}} = e^{\frac{0!s}{r!x}} \quad (5)$$

$$c \cdot \frac{0!s}{r!x} = \frac{0!s}{\frac{r}{c}x} \quad (6)$$

$$\left(\frac{1!s}{r!x} \right)^c = \frac{1!s/c}{(\frac{r}{c}x)!} \quad (7)$$

$$\frac{m!r}{r!x} = \frac{m!0}{-r!x} \quad (8)$$

The convergence radius of the master series in the complex plane is as follows:

For $r = 0$: The master series converges if $|sx| < 1$ where e is Euler's number (the base of the natural logarithm).

$$\begin{cases} |x| < \frac{1}{|s|e}, s \neq 0 \\ |x| < \infty, s = 0 \end{cases} \quad (9)$$

For $r \neq 0$: The convergence condition takes a more complex form:

$$\text{msr}(m; s; r; x) \text{ converges if } |r - s|^{r-s} |s|^{|s|} |x|^{|r|} < 1$$

$$\begin{cases} |x| < \frac{|r-s|^{r-s}}{|r-s|}, s \neq 0 \\ |x| < \frac{1}{|r|}, s = 0 \end{cases} \quad (10)$$

The key property of the method is that any trinomial equation $Ay^a + By^b + C = 0$ with $\text{Re}(a) > \text{Re}(b) > 0$ can be reduced to one of six canonical forms of master equations:



Table 1

Canonical Master Equations	
Original	Reflection
$Ay^a = -C - By^b \quad (1)$	$Cy^{-a} = -A - By^{b-a} \quad (6)$
$By^{b-a} = -A - Cy^{-a} \quad (2)$	$Ay^{a-b} = -B - Cy^{-b} \quad (5)$
$Cy^{-b} = -B - Ay^{a-b} \quad (3)$	$By^b = -C - Ay^a \quad (4)$

However, to obtain all roots of the original trinomial equation, in practice it is sufficient to use only three master equations. The remaining master equations are their mirror reflections: (1 and 6), (2 and 5), (3 and 4). They yield the same roots, sometimes for different values of the “root index” n .

The solution to this equation is identically equal to the corresponding master series:

$$py^r = q + xy^s \quad (11)$$

$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z}, \quad z = \frac{xv^s}{rq}, \quad v = \sqrt[r]{\frac{q}{p}} e^{\frac{(\arg(\frac{q}{p}) + 2\pi n)i}{r}}, \quad n \in \mathbb{Z} \quad (12, 13)$$

where p and q are non-zero. Note that with the rescaling $y \rightarrow \hat{y} = (q/p)^{1/r} y$ and $x \rightarrow \hat{x} = (p/q)^{s/r} qx$, we can write this equation as $\hat{y}^r = 1 + \hat{x}\hat{y}^s$. This means that, without any loss of generality, we can always take $p = q = 1$. We use the method of series generalization theorem [1] (analogous to Lagrange's theorem on the expansion of implicit functions) which allows us to express y as a power series in x . In Appendix A, we present a 5-step procedure to obtain the above solution.

The six forms of master equations form three mirror pairs that yield identical sets of roots. Thus, for a complete analysis of the original equation, it is sufficient to solve three independent master equations. For equations with four or more terms, the method provides a merge operation of master series, which allows constructing solutions in a similar analytical manner. For details on the merge operation, please refer to Reference [2]. The JavaScript implementation of the merge algorithm can be found in the Complex.js file (function maths) [3].

3. DETERMINING THE NUMBER OF ROOTS, THEIR VALUES, AND ANALYSIS OF THEIR BRANCHING

Before considering specific examples, it is necessary to clarify the fundamental question of choosing the branch of the complex logarithm when working with equations containing complex exponents. The key point is that to verify a root of the original equation of the form $F(y) = x$, only the principal branch of the logarithm ($k=0$) should be used. Using other branches ($k \in \mathbb{Z}$) is equivalent to solving a different equation, for example $|F(y)| = x$, which does not correspond to the original problem statement. Let's illustrate this with a simple example. Consider the equation $2\sqrt[2]{y} = 4$, which has a single obvious root $y = 4$. Let's verify this root strictly through the definition of the complex power:

$$\sqrt[2]{4} = e^{\frac{\ln|4| + \arg(4)i + 2\pi ik}{2}} \quad (14)$$

At $k=0$ we get $\sqrt[2]{4} = 2$, which satisfies the original equation. At $k=1$ we get $\sqrt[2]{4} = -2$, therefore this value is not a root. Thus, to verify the root of the original equation we must use only the principal branch ($k=0$). If we like to use other branches, we transform the original equation $2\sqrt[2]{y} = 4$ into the equation $|2\sqrt[2]{y}| = 4$. A similar approach applies to equations with arbitrary complex and irrational exponents. For example, the equation



$$y^\pi = 888 - 777i \quad (15)$$

Has three roots verifiable via the principal branch. The common misconception about an infinite number of roots arises from incorrect verification using all logarithm branches, which actually corresponds to solving the equation

$$|y^\pi| = |888 - 777i| \quad (16)$$

and not the original one.

The Master-J method is applied to trinomials of the form $Ay^a + By^b + C = 0$. During the analysis, such an equation is transformed into one of six canonical master equations $py^r = q + xy^s$, the solution of which is expressed through a master series. In this process, the complex logarithm is used at several stages: twice during solution construction (when calculating the reference point v and when raising it to the power v^s) and twice during the final verification of the root by substitution into the original equation. It is critically important that branching of the complex logarithm (selection of the parameter $n \in \mathbb{Z}$) is required only at one stage - when calculating the reference point v . At all other stages, including final verification, only the principal branch of the logarithm ($k=0$) is used. This ensures that the found solutions correspond exactly to the original equation and not its variations.

To demonstrate the capabilities of the method, we consider in Appendix B a trinomial with two irrational exponents, one of which is dynamically changed, rotating along the complex circle (parameter N from 0 to 15):

$$y^a + y^e = Q, \quad a = \pi e^{\frac{N\pi i}{8}} \quad (17, 18)$$

Therefore, even for "exotic" exponents, the method provides a complete answer. If the number of roots is finite, it provides all roots with any required accuracy. If the number of roots is infinite, the master method reveals the structure of the roots branch in the complex plane. The fact that one exponent is imaginary and the other is real does not necessarily mean there will be an infinite set of roots. For example, with $Q = 19333.68$, the number of roots is infinite, whereas for $Q = 19333.69$, it is finite.

4. CONCLUSION AND APPLICATION PROSPECTS

The Master-J method represents a novel approach in the solution of equations with complex exponents, offering the first complete analytical apparatus for this class of problems. The most significant achievement of the method is its ability not only to find roots but also to determine the nature of their set - finite or infinite.

The application prospects of the method cover various fields of science and technology. In fundamental mathematics, the method opens new possibilities for analyzing the properties of transcendental equations and their solutions. In physics, it can be applied to solve problems in quantum mechanics, field theory, and nonlinear dynamics, where equations with complex exponents may arise. In engineering disciplines, the method can be used to analyze the stability of systems with fractional derivatives and complex nonlinearities. In economics and finance, the method offers new approaches to modeling complex processes with nonlinear dependencies.

The universality of the Master-J method and its ability to provide exact analytical solutions make it a valuable tool for further research. Further development of the method may be directed towards expanding its application to equations with a larger number of terms, systems of equations, as well as developing efficient computational algorithms based on it.

Potential applications of the method extend to cryptographic authentication systems. The method's key advantage lies in the dynamical nature of the solution set: varying the equation's parameters not only changes the roots but can alter their quantity. This inherent unpredictability, combined with the analytical intricacy of complex exponents, provides a robust foundation for challenge-response protocols. Crucially, even with intercepted multiple responses, an adversary



cannot reconstruct the original equation since Viète's formulas are inapplicable to equations with complex exponents.

Acknowledgement:

I am grateful to A. D. Alhaidari for comments and suggestions that made significant improvements on the original manuscript.

APPENDIX A

In this Appendix, we give a 6-step procedure to obtain the Master-J solution of the trinomial equation $py^r = q + xy^s$. Rescaling x and y allow us to take $p = q = 1$.

Step 1: Preparing the Equation

The original equation: $y = (1 + rxy^s)^{\frac{1}{r}}$

Note: The form $1 + rxy^s$ (with the factor r) is chosen for universality. It is accomplished by rescaling the variable x in the original equation $y^r = 1 + xy^s$ as $x \rightarrow rx$. This allows the master series to solve various equations including the exponential case when $r=0$ ($y = e^{xy^s} = {}_{0:s}^{1:x}$) and the standard exponential when $r=0, s=0$ ($y = e^x = {}_{0:0}^{1:0}$). $s=0$ ($y = (1 + rx)^{\frac{1}{r}} = {}_{r:0}^{1:x}$). $y = (1 + rxy^s)^{\frac{1}{r}} = {}_{r:s}^{1:x}$. This unified approach enables solving many different equations through the same expansion. Note that one of the roots of entirely different equations coincides with the master series without modification of either parameters or the independent variable. To single out this particular root as the root of the same power series across different equations, we expand into a power series not the expression $1 + xy^s$, but the expression $1 + rxy^s$. This approach will enable us to create a comprehensive table of equations in the future where one of the roots is represented by a unified power series. Due to this approach, when using this universal power series to solve equations where $r \neq 0$, we must remember that for solving the familiar form $y^r = 1 + xy^s$, we should use the independent argument as $y = (1 + xy^s)^{\frac{1}{r}} = {}_{r:s}^{1:x/r}$.

We take the logarithm of both sides to simplify the power dependence:

$$\ln y = \frac{\ln(1 + rxy^s)}{r} \quad (19)$$

This is equivalent to:

$$\ln(1 + rxy^s) = r \ln y \quad (20)$$

Step 2: Series Expansion

We use the standard Taylor series expansion of the logarithm:

$$\ln(1 + z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \dots \quad (21)$$

Substitute $z = rxy^s$:

$$r \ln y = rxy^s - \frac{(rxy^s)^2}{2} + \frac{(rxy^s)^3}{3} - \dots \quad (22)$$

Divide by r :

$$\ln y = xy^s - \frac{r(xy^s)^2}{2} + \frac{r^2(xy^s)^3}{3} - \dots \quad (23)$$

Step 3: Representing "y" as a Power Series

Assume that y can be expressed as a power series in x :

$$y = 1 + \sum_{k=1}^{\infty} a_k x^k \quad (24)$$

Substitute this series into the equation for $\ln y$:

$$\ln(1 + \sum_{k=1}^{\infty} a_k x^k) = x(1 + \sum_{k=1}^{\infty} a_k x^k)^s - \frac{rx^2}{2} (1 + \sum_{k=1}^{\infty} a_k x^k)^{2s} + \dots \quad (25)$$

Step 4: Expanding $\ln y$ and y^s

Use the expansion of $\ln(1 + u)$ for small u :

$$\ln(1 + u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \dots \quad (26)$$



where $u = \sum_{k=1}^{\infty} a_k x^k$. Then:

$$\ln y = \sum_{k=1}^{\infty} a_k x^k - \frac{1}{2} (\sum_{k=1}^{\infty} a_k x^k)^2 + \frac{1}{3} (\sum_{k=1}^{\infty} a_k x^k)^3 - \dots \quad (27)$$

For y^s use the binomial expansion:

$$y^s = (1 + \sum_{k=1}^{\infty} a_k x^k)^s = 1 + s \sum_{k=1}^{\infty} a_k x^k + \frac{s(s-1)}{2} (\sum_{k=1}^{\infty} a_k x^k)^2 + \dots \quad (28)$$

Step 5: Equating Coefficients

Substitute the expansions into the equation:

$$\sum_{k=1}^{\infty} a_k x^k - \frac{1}{2} \left(\sum_{k=1}^{\infty} a_k x^k \right)^2 + \dots = x \left(1 + s \sum_{k=1}^{\infty} a_k x^k + \dots \right) - \frac{rx^2}{2} (1 + 2s \sum_{k=1}^{\infty} a_k x^k + \dots) + \dots \quad (29)$$

Comparing coefficients of like powers of x on both sides of the equation yields the coefficients a_k , giving the series:

$$y = 1 + x + \frac{2s-r+1}{2} x^2 + \frac{9s^2-9sr+2r^2+6s-3r+1}{3!} x^3 + \dots \quad (30)$$

This series coincides with the master series $\frac{1 \cdot s}{r \cdot x}$, presented in the article:

$$\frac{1 \cdot s}{r \cdot x} = 1 + x + \frac{1+2s-r}{2} x^2 + \frac{(1+3s-r)(1+3s-2r)}{3!} x^3 + \dots \quad (31)$$

APPENDIX B

In this Appendix, we give the Master-J solutions of the equation $y^a + y^e = Q$, $a = \pi e^{\frac{N\pi i}{8}}$ for $Q = 888 - 777i$ and for several values of N

We always transform the original equation into three master equations (see Table 1) and search for roots in all three selected master equations using the same method. If roots of the master equation exist, they are determined through a continuous range of base points $v \square$. Until a formula for determining the boundaries of this n range is found, we proceed as follows. First, we find the boundaries of such n values for which the roots are verified in the original equation only through the principal branch of the complex logarithm. Besides verification, the boundaries can be determined by the following conditions: divergence of the series if $R = |r-s| |s|^{|s|} |z|^{|r|} \geq 1$, and root repetition - if previously found roots are obtained at new values of n . In cases where one exponent is real and the other is purely imaginary (not complex but specifically imaginary), one of the boundaries might be absent; in this case, we must show several roots from the infinite set starting from the first one, as they are naturally ordered by modulus.

$N=0$: Finite Number of Roots with Real Exponents

We transform the equation $y^\pi + y^e - (888 - 777i) = 0$ into master equation (1), see Table 1.

$$y^\pi = (888 - 777i) - y^e \quad (32)$$

Apply the master method

$$py^r = q + xy^s \quad (33)$$

$$y = v \cdot \frac{1 \cdot s}{r \cdot z}, \quad z = \frac{xv^s}{rq}, \quad v = e^{\frac{\ln|q/p| + (\arg(\frac{q}{p}) + 2\pi n)i}{r}}, \quad n \in \mathbb{Z} \quad (34, 35, 36)$$

Substitute the values from master equation (1) into the master method parameters.

$$p = 1, r = \pi, q = 888 - 777i, x = -1, s = e$$

$$n=-1, y=-5.9570521363573448949-6.4972651375071501499i$$

$$n=0, y=8.292133856287354895-2.011891769278423965i$$

$$n=1, y=-2.3138498235199130471+8.3953403281683450881i$$

For master equation (2), see Table 1, $y^{e-a} = -1 + Qy^{-a}$, the series diverges for all " n "



$$R = |r - s|^{r-s} |s|^{s|z|} |z|^{r|z|} = 15877 > 1 \quad (37)$$

For master equation (4), see Table 1, $y^e = Q - y^a$, the series also diverges for all "n"

$$R = |r - s|^{r-s} |s|^{s|z|} |z|^{r|z|} = 33.39 > 1 \quad (38)$$

N=1: Finite Number of Roots with Complex Exponents

Equation:

$$y^{\pi e \frac{\pi i}{8}} + y^e - Q = 0 \quad (39)$$

Transform into master equation (1), see Table 1.

$$y^{\pi e \frac{\pi i}{8}} = Q - y^e \quad (40)$$

Apply the master method

$$py^r = q + xy^s \quad (41)$$

$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z}, \quad z = \frac{xv^s}{rq}, \quad v = e^{\frac{\ln|\frac{q}{p}| + (\arg(\frac{q}{p}) + 2\pi n)i}{r}}, n \in Z \quad (42, 43, 44)$$

Substitute the values from master equation (1) into the master method parameters.

$$p = 1, r = \pi e \frac{\pi i}{8}, q = 888 - 777i, x = -1, s = e, n = 0 \quad (45)$$

$$v = e^{\frac{\ln|888-777i| + \arg(888-777i)i}{\pi e \frac{\pi i}{8}}} = 3.50197 - 6.44i \quad (46)$$

$$z = \frac{-v^e}{\pi e \frac{\pi i}{8} (888-777i)} = .05172 + .03178i \quad (47)$$

$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z} = 3.9445 - 6.6828i \quad (48)$$

Recall that $\frac{1}{r} \cdot \frac{s}{z}$ is a power series with a limited convergence region, not a function working in the entire complex domain.

$$\begin{aligned} \frac{m}{r} \cdot \frac{s}{x} = m + x + \frac{x^2}{2}(m + 2s - r) + \frac{x^3}{3!}(m + 3s - r)(m + 3s - 2r) \\ + \frac{x^4}{4!}(m + 4s - r)(m + 4s - 2r)(m + 4s - 3r) + \dots \end{aligned} \quad (49)$$

Now verify the found root in the master equation $y^{\pi e \frac{\pi i}{8}} = Q - y^e$

The root $y = 3.9445 - 6.6828i$, indeed satisfies the original equation. Continue to n=-1.

$$n = -1, \quad v = e^{\frac{\ln|888-777i| + \arg(888-777i)i - 2\pi i}{\pi e \frac{\pi i}{8}}} = -3.3 - .747i \quad (50)$$

$$z = \frac{-v^e}{\pi e \frac{\pi i}{8} (888-777i)} = -.0018 + .007i \quad (51)$$

$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z} = -3.31 - .77i \quad (52)$$

Verify this root in the master equation $y^{\pi e \frac{\pi i}{8}} = Q - y^e$

The root $y = -3.31 - .77i$, indeed satisfies the original equation. Note: numerical results in this article are rounded for compactness. For thorough verification, use unrounded values.

We must check for roots in all three master equations (1 or 6), (2 or 5), and (3 or 4). Take the (2) master equation from the original, see Table 1.

$$By^{b-a} = -A - Cy^{-a} \quad (53)$$

$$y^{e-a} = -1 + Qy^{-a} \quad (54)$$

Apply the master method

$$py^r = q + xy^s \quad (55)$$



$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z}, \quad z = \frac{xv^s}{rq}, \quad v = e^{\frac{\ln|\frac{q}{p}| + (\arg(\frac{q}{p}) + 2\pi n)i}{r}}, \quad n \in \mathbb{Z} \quad (56)$$

Substitute the values from master equation (2) into the master method parameters.

$$p = 1, r = e - \pi e^{\frac{\pi i}{8}}, q = -1, x = 888 - 777i, s = -\pi e^{\frac{\pi i}{8}} \\ n = -2 \\ v = e^{\frac{\ln|-1| + \arg(-1)i - 4\pi i}{e - \pi e^{\frac{\pi i}{8}}}} = 820.885 + 1955.7i \quad (57)$$

$$z = \frac{(888 - 777i)v^{-\pi e^{\frac{\pi i}{8}}}}{-(e - \pi e^{\frac{\pi i}{8}})} = -.000000507 - .0000007i \quad (58)$$

$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z} = 820.8862 + 1955.66671i \quad (59)$$

This root, when substituted into the original equation, is verified. The roots of the fifth master equation for $n=-3$ and $n=-4$ are also verified. For $n=-5$ and beyond, the roots are not verified in the original equation using the principal branch of the logarithm, meaning they are not roots of the original equation.

$$\text{master equation (4)} \quad y^{\pi e^{\frac{\pi i}{8}}} = Q - y^e$$

This equation yields a single root at $n=1$. The others will not be verified using the principal branch of the complex logarithm.

$$y^{\pi e^{\frac{\pi i}{8}}} + y^e = 888 - 777i \quad (60)$$

Thus, the Master-J method strictly proves that the equation $y^{\pi e^{\frac{\pi i}{8}}} + y^e = 888 - 777i$ has exactly six roots. However, complex exponents alone do not determine the number of roots, as in algebraic equations with integer exponents. For example, with $Q=0.001$, this equation has 7 roots.

N=4: Infinite Number of Roots

Consider the case where the number of roots is truly infinite. One exponent is real, the other is purely imaginary (real part equals zero):

$$y^e + y^{\pi i} - Q = 0 \quad (61)$$

$$Q = 888 - 777i \quad (62)$$

Transform into master equation (1) (see Table 1):

$$y^e = Q - y^{\pi i} \quad (63)$$

Apply the master method

$$py^r = q + xy^s \quad (64)$$

$$y = v \cdot \frac{1}{r} \cdot \frac{s}{z}, \quad z = \frac{xv^s}{rq}, \quad v = e^{\frac{\ln|\frac{q}{p}| + (\arg(\frac{q}{p}) + 2\pi n)i}{r}}, \quad n \in \mathbb{Z} \quad (65, 66, 67)$$

Substitute the values from master equation (1) into the master method parameters.

$$p = 1, r = e, q = 888 - 777i, x = -1, s = \pi i \\ n = 0 \quad (68)$$

$$v = e^{\frac{\ln|888-777i| + \arg(888-777i)i}{e}} = 13.02 - 3.5i \quad (69)$$

$$z = \frac{-v^{\pi i}}{e(888-777i)} = .000617 - .000362i \quad (70)$$

$$y = v \cdot \frac{1}{e} \cdot \frac{\pi i}{z} = 13.0297 - 3.533i \quad (71)$$

Then we obtain the second root of the first master equation at $n=1$:

$$n = 1$$



$$y = -6.18497 + 11.99i \quad (72)$$

Now transform the original equation into master equation (2).

$$y^{\pi i - e} = -1 + Qy^{-e} \quad (73)$$

Here there is only one root:

$n = 2$: $y = -14.694474242164030919 - 12.569844656322447529i$

The most interesting case is master equation (4)

$$y^{\pi i} = 1 - Qy^e \quad (74)$$

Here we obtain an infinite number of roots for "n" from $-\infty$ to $n=1$.

$n = \dots$

$n = -7, y = -.00000041627385621645907377-.00000051405285089600779712i$

$n = -6, y = -.0000030758708761016074812-.0000037983653530858345787i$

$n = -5, y = -.000022727782456581737168-.000028066334678185757061i$

$n = -4, y = -.00016793685957596076555-.00020738372142847451195i$

$n = -3, y = -.0012408948764620104223-.0015323699516353133188i$

$n = -2, y = -.0091690418160200212443-.011322767528988349131i$

$n = -1, y = -.067750498115986331295-.083664550958259858241i$

$n = 0, y = -.50049986982281044678-.61817908960892751252i$

$n = 1, y = -3.5224985137151535401-4.5133344215533415888i$

Note that the roots are ordered by modulus.

References:

1. Henrici, P. "Applied and Computational Complex Analysis", Volume 1, Wiley, 1974, Section 1.9 (The Lagrange-Bürmann Expansion); Whittaker, E.T. and Watson, G.N. "A Course of Modern Analysis", Cambridge University Press, Chapter 5 (The Lagrange Expansion).
2. MASTER-J: UNIVERSAL ANALYTICAL METHOD FOR SOLVING EQUATIONS AND ITS APPLICATION IN SCIENCE AND FINANCE / P. V. Berezin, A. V. Berezin, S. V. Berezin, A. V. Gruzlov // Flagman nauki. - 2025. - No. 7(30). DOI 10.37539/2949-1991.2025.30.7.011 https://flagmannauki.ru/files/730-Berezin_Sergey_Viktorovich_4135_2.pdf
3. Master-J Laboratory. DOI 10.5281/zenodo.15633781

