

**Меликов Эльчин Адил**, Доцент,  
Азербайджанский Государственный Университет  
Нефти и Промышленности  
Melikov Elchin Adil, associate professor,  
Azerbaijan State University of Oil and Industry

**Агавердиев Тебрик Рантик**,  
Азербайджанский Государственный Университет  
Нефти и Промышленности  
Agaverdiyev Tebrik Rantik, master,  
Azerbaijan State University of Oil and Industry

**ПОСТРОЕНИЕ МАТЕМАТИЧЕСКОЙ МОДЕЛИ  
И ЗАДАЧА ОПТИМИЗАЦИИ ФУНКЦИОНИРОВАНИЯ  
УСТАНОВКИ ГИДРОГЕНИЗАЦИОННОЙ ОЧИСТКИ МАСЕЛ  
CONSTRUCTION OF A MATHEMATICAL MODEL AND THE PROBLEM  
OF OPTIMIZING THE FUNCTIONING OF AN OIL HYDROGENATION  
PURIFICATION INSTALLATION**

**Аннотация.** В статье на основе всестороннего исследования и анализа процесса гидрогенизационной очистки масел рассматривается проблема построения модели и последующей оптимизации исследуемого процесса на рассматриваемой установке. Целью процесса является снижение содержания серы, азота и кислорода в маслах, что улучшает их эксплуатационные характеристики. В работе выбрана нелинейная регрессионная модель второго порядка. Выходной переменной выбрана вязкость очищенного масла при 50 °С, которая выступает индикатором качества получаемого готового продукта. На основе полученной модели сформулирована и решена задача оптимизации.

**Abstract.** This article, based on a comprehensive study and analysis of the process of oil hydrogenation purification, examines the problem of constructing a model and subsequently optimizing the studied process at the considered installation. The purpose of the process is to reduce the sulfur, nitrogen, and oxygen content in oils, which improves their performance characteristics. A second-order nonlinear regression model is selected in this study. The output variable is the viscosity of the purified oil at 50°C, which serves as an indicator of the finished product quality. Based on the resulting model, an optimization problem is formulated and solved.

**Ключевые слова:** Очистка масел, математическая модель, задача оптимизации, технологический процесс, концентрация водорода.

**Keywords:** Oil purification, mathematical model, optimization problem, technological process, oil viscosity, hydrogen concentration.

As is known, to control and optimize the functioning of complex technological objects, a mathematical model is first of all necessary. In addition, the model helps to predict the future behavior of an object under various conditions, simplifying its understanding and research without working directly with the real object. To construct a model, important stages include studying the processes occurring at the installation, determining the technological parameters that have a significant impact on the technological process, selecting the type of model, and collecting statistical information that defines the actual state of the technological object over time [1-5].



The presented article is devoted to a thorough study and comprehensive analysis of the technological processes occurring at one of the important and relevant installations in both petrochemistry and oil refining, namely, a technological installation for the hydrogenation purification of oils.

Hydrogenation refining of oils is a complex process of chemical hydrocarbons modification used to improve the quality of petroleum products (oils, fuels) and to obtain valuable chemicals (e.g., cyclohexane) using hydrogen and catalysts under pressure. The process of cleaning contaminated oils occurs in the presence of hydrogen-containing gas, and the contaminants are nitrogen, oxygen, and sulfur-containing compounds. This process aims to reduce the amount of contaminants in the oil, improve its quality characteristics (flash point, density, base number, viscosity, pour point, etc.), and improve its chemical resistance [6]. The process is as follows: contaminated oil is fed to be mixed with hydrogen-containing gas, then it is fed into heating apparatuses, heating up to a temperature in the range of 260-350 °C. The heated gas-feed mixture is then sent to the reactor. Here in the reactor the main purification reactions take place. The resulting mixture is then sent to a separator and then to a stripping column. In this column, the resulting mixture of purified oil is separated from its side ingredients. The purified oil is then delivered to the finished goods park. The supply of hydrogen-containing gas for mixing, its hydrogen concentration, and the values of other process parameters are carefully regulated (Figure 1).

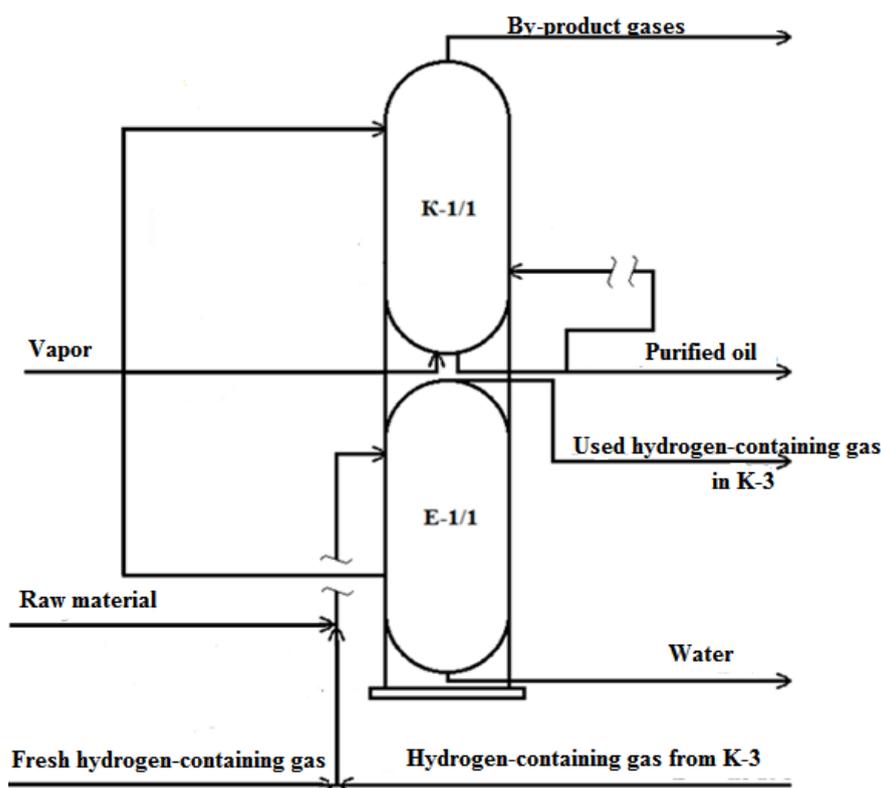


Figure 1. The principal scheme of the installation

The main input parameters of the process under study that most strongly influence the achievement of the selected optimality criterion are the following:  $z_1$  is the raw material consumption at the entrance to the installation ( $m^3/h$ );  $z_2$  is the consumption of hydrogen-containing gas supplied for mixing ( $m^3/h$ );  $z_3$  is the pressure at the entry point of the stripping column K-1/1 ( $kPa$ );  $z_4$  is the vapor usage in the stripping column K-1/1 ( $kg/h$ );  $z_5$  is the stripping column K-1/1 bottom temperature



(°C);  $z_6$  is the amount of hydrogen in the hydrogen-containing gas mixed with the raw material ( $m^3/h$ ). The viscosity of the purified oil at 50 °C is also measured to assess its quality.

Even though one of the main apparatuses in the installation is a reactor, its parameters are not controlled due to the complexity of the processes occurring in it and a fairly large number of factors influencing the course of reactions, including: catalyst activity, reaction time, as well as other indicators that can take on different indicators depending on the position in time and space. Given the above considerations, direct intervention in the reactor processes is impossible, and the measured process variables are not incorporated into the model. The model of this installation is selected in the regression model form. Most often, to simplify calculations, linear regression models are used, but, given that most processes in oil refining, petrochemistry, and chemistry are in reality complex, multi-component, non-stationary, it would be more correct to choose a model of the installation under study in the form of a non-linear regression equation. We use the following nonlinear equation:

$$y = a_0 + a_1z_1 + a_{11}z_1^2 + a_2z_2 + a_{22}z_2^2 + a_3z_3 + a_{33}z_3^2 + a_4z_4 + a_{44}z_4^2 + a_5z_5 + a_{55}z_5^2 + a_6z_6 + a_{66}z_6^2 \quad (1)$$

To find the unknown coefficients of the regression equation (1) least squares method. The model is built on the basis of real statistical data collected at a functioning installation. The numerical values calculation for the coefficients of the nonlinear regression model is performed in the *Microsoft Office Excel* program. Taking into account the found coefficients obtain a model of the following type:

$$y = 35.150453 - 0.019473 * z_1 + 0.001548 * z_2 - 0.000151 * z_3 + 0.005633 * z_4 - 0.001211 * z_5 - 0.000002 * z_6 - 0.002925 * z_1^2 - 0.000007 * z_2^2 - 0.000201 * z_3^2 - 0.000175 * z_4^2 - 0.000051 * z_5^2 - 0.000004 * z_6^2 \quad (2)$$

To test the adequacy of the model, the multiple correlation coefficient and Fisher's criteria test are calculated. After calculating the necessary determinants, we obtain the following value for the multiple correlation coefficient:  $R = 0,983546$ . The calculations reveal a strong correlation among the chosen parameters within the resulting model. Next, the statistical reliability of the regression equation is assessed using the Fisher criterion. The processes occurring in this installation are aimed at reducing the purified oils' viscosity, which determines the formulation of the optimization problem as the minimization of the objective function. Equation (2) is chosen as the objective function, and the constraints system is represented as:

$$\begin{cases} 18 \leq x_1 \leq 22 \\ 7500 \leq x_2 \leq 8500 \\ 95 \leq x_3 \leq 105 \\ 120 \leq x_4 \leq 140 \\ 160 \leq x_5 \leq 180 \\ 6250 \leq x_6 \leq 7250 \end{cases} \quad (3)$$

To solve an optimization problem in the *MATLAB*, the main step is writing a script that executes the procedures necessary to find the optimum. Then we go to the *MATLAB* command window and enter the name assigned to the script into the command line. After entering the name, we press Enter, and the optimal values of the function being minimized are calculated under the specified constraints. The calculation result is:



```
>> myfun
```

```
Local minimum found that satisfies the constraints.
```

```
Optimization completed because the objective function is non-decreasing in  
feasible directions, to within the default value of the optimality tolerance,  
and constraints are satisfied to within the default value of the constraint tolerance.
```

```
<stopping criteria details>
```

```
ans =
```

```
1.0e+03 *
```

```
0.0220 7.5000 0.1050 0.1400 0.1800 7.2500
```

Figure 2.

By substituting these values into the equation of the mathematical model, we obtain the value corresponding to the function minimum:

$$35.150453 - 0.019473 * 22 + 0.001548 * 7500 - 0.000151 * 105 + 0.005633 * 140 - 0.001211 * 180 - 0.000002 * 7250 - 0.002925 * 22 * 22 - 0.000007 * 7500 * 7500 - 0.000201 * 105 * 105 - 0.000175 * 140 * 140 - 0.000051 * 180 * 180 - 0.000004 * 7250 * 7250 = 31.562971 \approx 31.56$$

The obtained viscosity value at 50°C, while maintaining the values of the regulated parameters equal to those specified in the system (3), is  $y=31.56$ . Thus, based on a comprehensive study of the processes occurring at the oil hydrogenation purification installation, established that it is important to maintain the hydrogen concentration in the hydrogen-containing gas in accordance with the existing regulations.

*Список литературы:*

1. Guseinov I.A., Khanbutaeva N.A., Melikov E.A., Efendiev I.R. Models and Algorithms for a Multilevel Control Systems of Primary Oil Refinery Installations. Journal of Computer and Systems Sciences International, Pleiades Publishing, Ltd., 2012, Vol. 51, No. 1, pp. 138-146. <https://doi.org/10.1134/S1064230711060098>

2. Safarova A.A., Melikov E.A., Magerramova T.M. Design of a cascade controller based on a predictive model. Fourth International Conference on Digital Technologies, Optics, and Materials Science (DTIEE 2025), Proceedings of SPIE, Vol. 13662, 136620H, 2025. <https://doi.org/10.1117/12.3072311>

3. Melikov, E.A., Magerramova, T.M., Safarova, A.A. Logical-Linguistic Model for Reactor Cleaning from Impurities. 15th International Conference on Applications of Fuzzy Systems, Soft Computing and Artificial Intelligence Tools – ICAFS-2022. Lecture Notes in Networks and Systems, vol 610. Springer, Cham., pp. 321-329. [https://doi.org/10.1007/978-3-031-25252-5\\_44](https://doi.org/10.1007/978-3-031-25252-5_44)

4. Melikov E.A., Safarova A.A., Magerramova T.M. Features of modelling in automation for the primary oil refining technological process. In Third International Conference on Digital Technologies, Optics, and Materials Science (DTIEE 2024), SPIE, Vol. 13217, pp. 125-130. <https://doi.org/10.1117/12.3035889>

5. Safarova A.A., Melikov E.A., Magerramova T.M. Principles of modeling and optimal control of a fraction purification reactor from micro impurities, III International scientific and practical conference “Technologies, materials science and engineering”, AIP Conference Proceedings, Vol. 3243, Issue 1, 020014, 2024. <https://doi.org/10.1063/5.0247869>



6. Aliev I., Melikov E.A. Experimental method for constructing a mathematical model of a hydrogenation oil cleaning installation. Scientific Journal of Italy “Annali d’Italia”, No. 39, 2023, pp. 78-82. <https://doi.org/10.5281/zenodo.7568947>.

